Towards Practical Design Rules for Quantum Communications and Quantum Imaging Devices

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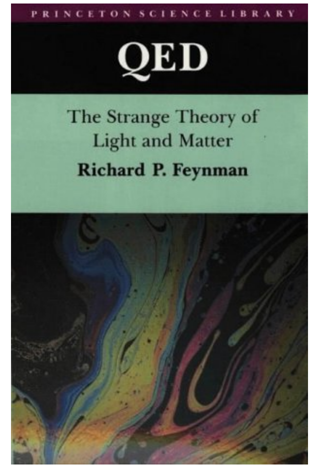
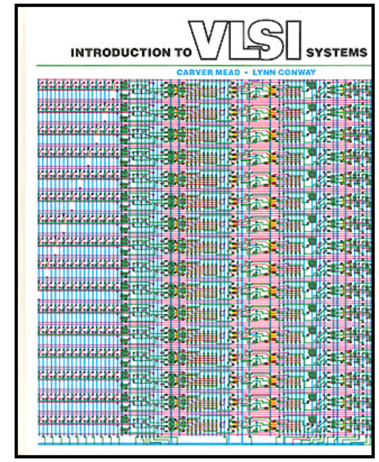
Main Results

1. First to introduce **quantum design rules** (§3 of our paper)
2. Rules based on Feynman's **quantum path integral** (QPI)
3. Derive **classical optics** from QPI
4. Show how current quantum optics methods are **unified** by QPI
5. Entangled biphotons as QPI **loops** (§4.5)
6. Complementarity paradox resolved by applying our quantum design rules (§5)
7. Proposed **bifurcation** of coherent light in a **lens**

Inspirations

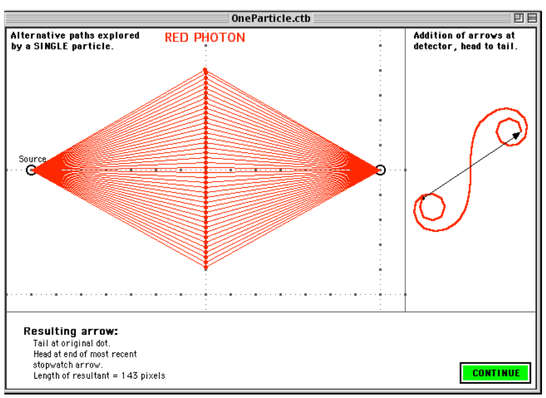


The photon rules! ☺



Rules map science to technology

Plethora of Paths

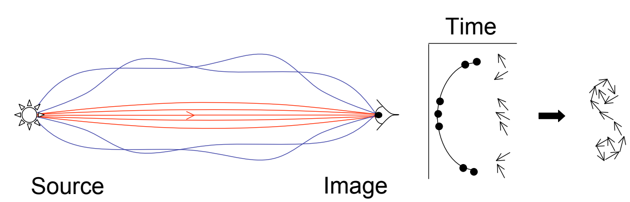


Taylor & Styer pedagogic QPI tool shows development of Cornu spiral. Download available at `www.eftaylor.com/download.html\#quantum`

QPI paths between **mono chromatic** source and detector. Phase of each path represented as a **complex vector** with associated photon **frequency** ω. Angle of each final vector determined by **time of flight**. Vectors on extreme paths tend to **scroll up** due to their different final phase angles. When added vectorially they **cancel** each other.

Tubes of Light

Need more than the Fermat path to get correct intensity at detector.



Each photon path functional $\phi[x(t)]$ is a complex *vector* ($\nearrow$):

$$\phi[x(t)] = A \exp\left(\frac{i}{\hbar}\int L(\mathbf{x}, \dot{\mathbf{x}})dt\right)$$

Complete photon propagator from *source* to *detector*:

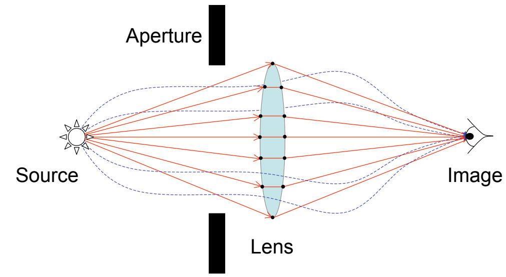
$$G(d|s) = \sum_{All\ paths} \phi[x(t)] \qquad \text{(Cornu spiral)}$$

Design Rules

(See §3 of our paper)

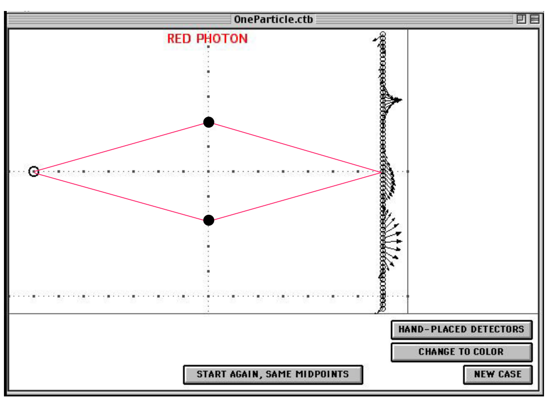
1. The physical photon is a *quantum entity*, not a microscopic "billiard ball".
2. Photons only interact with electrons via *events*, not via other photons. The final event is detection.
3. Between events, a photon acts like a free particle. Different QPI paths $\phi[x(t)]$ determined by their phase $S[x(t)]/\hbar$.
4. Introducing any material into an optical device introduces electrons which can cause new events.
5. A photon that undergoes an intermediate event starts a new QPI path as a different photon.
6. Successive path segments between source and detector induce a *product* of QPI propagators: $\phi_d[x(t)] \times \phi_x[x(t)] \times \phi_y[x(t)] \times \ldots \times \phi_s[x(t)]$.
7. Sum of all possible photon paths $G(d|s) = \sum \phi[x(t)]$ determines outcome of physical photon. Paths with similar phases will reinforce at detector, others will tend to cancel.
8. Only $\overline{G(d|s)}\, G(d|s)$ (*norm*) can be compared with physical measurement.

Convergent Lens



1. All possible photon paths have equal probability.
2. Lens is matter ⇒ interaction events. (**Rule 2**)
3. Resultant vector due to *isochronous* QPI paths. (**Rule 7**)
4. Only **isochronous** paths have aligned vectors. Other vectors cancel.
5. *Index of refraction* relegated to the historical scrapbook.

QPI Paths for Pure Interference

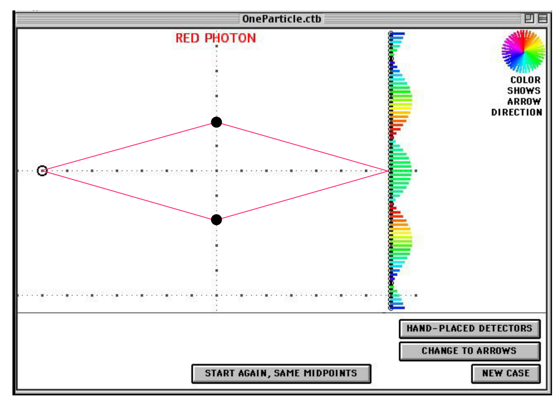


Consider any 2 QPI paths from each source (2 dots on center vertical) to screen:

$$\phi_1 = Ae^{-i\omega t_1} \qquad \text{and} \qquad \phi_2 = Ae^{-i\omega t_2}$$

where A is a constant spatial factor. The QPI calculator tool shows vectors belonging to many such paths at the image plane.

Pure Interference Pattern



Path vectors summed and modulus-squared to produce intensity fringes.

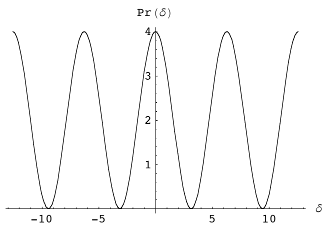
The orientation of the vectors is color-coded (wheel at top right).

Phase difference $\delta(y) = 2\pi \times$ (path difference at y) / λ.

QPI Probability Function

By **Rule 8** the probability is calculated from the norm.

$$
\begin{aligned}
Pr_\gamma(\delta) &= |G(d|s)|^2 \\
&= \overline{(\phi_1+\phi_2)}(\phi_1+\phi_2) \\
&= 2A^2(1+\cos\delta) \\
&= 4A^2\cos^2\left(\frac{\delta}{2}\right)
\end{aligned}
$$

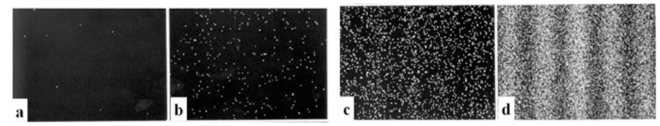


is the quantity to be compared with intensity measurement.

- Corresponds to Young-type pure interference fringes
- Identical to intensity from classical EM theory (Born & Wolf)

What happens if there is only a **single** photon?

Single Photon Interference



Tonomura `www.hqrd.hitachi.co.jp/em/doubleslit.cfm` 6000 electrons over 20 min.

Similar photon movie available at `idol.union.edu/~malekis/QM2004/qm_heis3.htm`

- Classical wave theory:

$$I(\delta) = 2\langle \mathbf{E}^2\rangle(1+\cos\delta)$$

Intensity of EM radiation makes no sense!

- Quantum mechanically:

$$Pr_\gamma(\delta) = 2A^2(1+\cos\delta)$$

Probability of finding **single** photon at position y on screen.

QPI for Diffraction

Interference uses **single**-segment paths: *aperture* → *screen*.

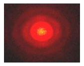
Diffraction includes **double**-segmented QPI paths:

source → *aperture* → *screen*

Rule 6 states: $G(d|s) = G(d|a)\times G(a|s)$. Examples:

$$G(d|s) \equiv \sum \Box \times e^{-i\omega t}\times e^{-i\omega(t-\tau)} \to \textbf{Sinc function}$$

$$G(d|s) \equiv \sum \bigcirc \times e^{-i\omega t}\times e^{-i\omega(t-\tau)} \to \textbf{Airy disk}$$

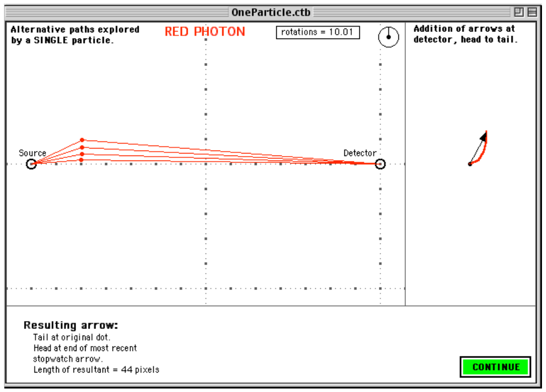


Sum over QPI paths acts like corresponding Fourier transforms

$\mathcal{F}$(aperture) followed by $\mathcal{F}^{-1}$(transformed aperture)

QPI for Rectangular Aperture

Squeezing light through aperture means fewer paths to cancel ⇒ **spreading**.



QPI vectors form polygon. In continuum limit → **circular** arc of length $s = r\delta$. Resultant vector length:

$$
\begin{aligned}
&= 2r\sin(\delta/2) \\
&= s\,(2/\delta)\sin(\delta/2) \\
&= s\,\textbf{sinc}(\delta/2)
\end{aligned}
$$

Propagator and probability:

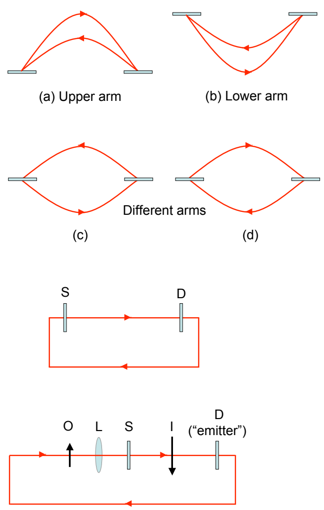
$$
\begin{aligned}
G(d|s) &\sim \mathrm{sinc}(\delta/2) \\
\Pr(\delta) &\sim \mathrm{sinc}^2(\delta/2)
\end{aligned}
$$

Will use $\Pr(\delta)$ in slide 20 to derive **Heisenberg uncertainty principle**.

Another result relegated to the historical scrapbook.

QPI for Entanglement & Ghosting

(See §4.5 of our paper)

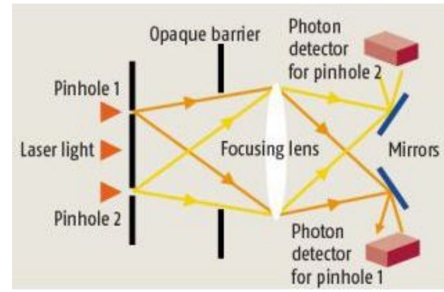


QPI paths for entangled photons in each interferometer arm. Correlated photons traverse together (a & b) or traverse each arm separately (c & d). Each QPI propagator is multiplied due to their intrinsic **correlation**. But this is the same as **Rule 6**! So each photon can be regarded as forming a biphoton **loop** between source and detector.

Diagram at left is topologically equivalent of the biphoton **loop** in diagram (d). If an object O and a convergent lens L are placed to the left of the biphoton source S and the detector at D is also regarded as an "emitter", the arrangement bares a striking resemblance to the *advanced wave* model of **quantum ghosting**.
(cf. Klyshko, Lvovsky, Shih et al.)

Interferometer Paradox

Recent (unpublished) claim: *Can determine* **which-way** *photon information in the presence of* **interference**, *using a modified Young-Wheeler interferometer.*



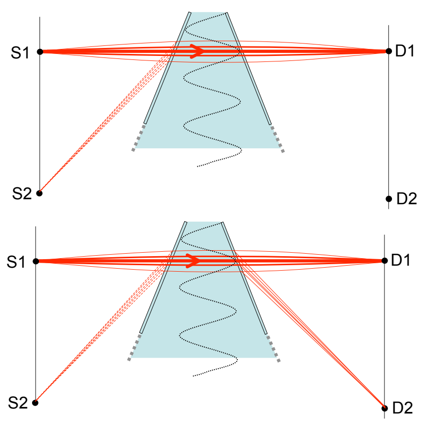
- Laser wavelength λ: 650 nm.
- Optical path length: 5 m.
- Pinhole diameter: 0.25 mm.
- Aperture diameter: 21 cm.
- Lens diameter: 3 cm.
- Lens distance: 4.2 m.
- Focal length: 100 cm.
- Source separation: 2 mm.
- Detector separation: 0.6 mm.

Guessing claim is wrong (violates **Bohr complementarity**) is easy. **Proving** it wrong is much harder. (BTW: Einstein's "photon" violated Maxwell's theory in 1905 and took 15 years to be validated experimentally).

We apply our **quantum design rules** to expose the **subtle** error.

Coherent QPI Bifurcation

(See §5.3 of our paper)



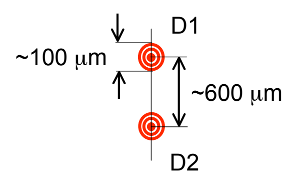
Lens cross-section. **Normally** S_1 not imaged at D_1. QPI phases cancel at D_1 due to **Rule 7**. But *normally* ⇒ **incoherent** light. Interferometer has twin **coherent** sources. Paths passing through interference **maxima** in the lens will now reinforce at D_1.

Paths passing through interference **minima** in the lens will cancel at D_1 and elsewhere. Paths passing in between these interference **extrema** will have differing phases and "refract" to D_2 in the usual way for incoherent light.

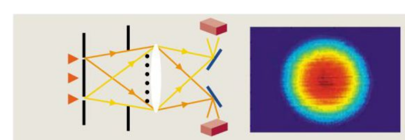
This refractive **bifurcation** is a property of coherent light, not the lens.

If you assume classical optics (geometric rays, Snell's law) you get it wrong.

Experimental Tests



Reported dimensions of image spots.



Rings shown here are from a computer generated image spot.

- Dual image spots due to symmetry of transfer function at lens between source and image planes.
- D_1 and D_2 should each see a "target" due to maximum and minimum coherency in the lens.
- Therefore photons arrive from **both** sources (Bohr lives!).
- In reality these rings are smeared out due to continuous nature of phase coherence.
- Application for entangled biphoton tagging?

What Kind of Photon Is This?

Light is relativistic ($v \simeq c$), massless and polarized.

Usual QPI is a Green's function for the Schrödinger wave equation: (Action for non-relativistic Newtonian particle with mass m_e !!!)

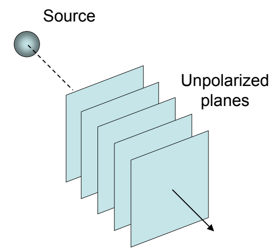
Preceding slides used semi-classical approximation for photon:

- Our QPI is a Green's function for **Klein-Gordon** wave equation ≡ relativistic, massless, **un**-polarized **scalar** particle
- Real photon QPI is a Green's function for **Maxwell-Lorentz** equation ≡ relativistic, massless, polarized **vector** particle

Klein-Gordon photon is logically equivalent to **Huygens-Fresnel** scalar **wave** theory. Same vector superposition rules as QPI. (Keep this in mind when reading Feynman's book)

Make It Plane

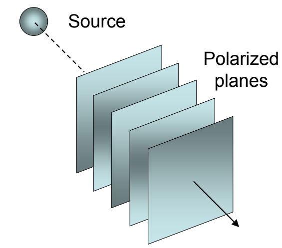
Scalar Plane Wave



- Huygens-Fresnel wave theory.
- Plane wave ≡ free photon.
- $\phi(x_\mu)$ scalar field.
- Spin 0 ⇒ no polarization.
- $\phi(x_\mu)$ solves Klein-Gordon:

$$(\partial_x^2 - \frac{1}{v^2}\partial_t^2)\,\phi = 0.$$

Vector Plane Wave



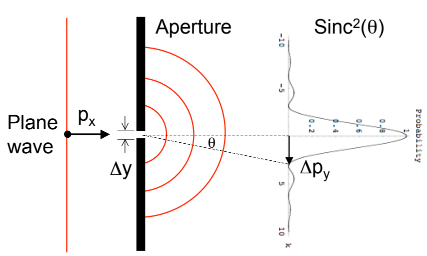
- Maxwell EM wave theory.
- Plane wave ≡ free photon.
- $\mathbf{A}_\mu$ vector field.
- Spin 1 ⇒ polarized **E**-field.
- $\mathbf{A}_\mu$ solves Maxwell-Lorentz:

$$(\partial_x^2 - \frac{1}{c^2}\partial_t^2)\,\mathbf{A}_\mu = 0.$$

Heisenberg for Free!

Klein-Gordon plane wave represents a **free** photon with momentum p_x.

First **minimum** in $\Pr(\theta) \sim \mathrm{sinc}^2(\theta)$ [from slide 13] occurs when the path length from one edge of aperture differs by λ from the other edge.



Transverse momentum:

$$\begin{aligned}\Delta p_y &= p_x\, sin\theta \\ &= p_x \frac{\lambda}{\Delta y} \\ &= \frac{h}{\lambda}\,\frac{\lambda}{\Delta y} \\ \Delta p_y.\Delta y &= h\end{aligned}$$

Attempting to localize the photon with a narrow aperture causes it to diffract and spread its momentum by Δp_y. This is Heisenberg's uncertainty principle:

$$\boxed{\boldsymbol{\Delta y.\Delta p_y \geq h}}$$

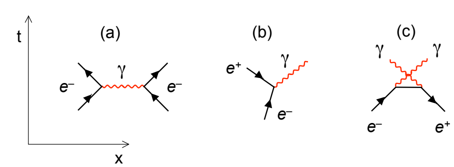
Feynman QED Propagators

- **Photon** (Maxwell-Lorentz boson wave equation)

$$\begin{aligned}\partial_\mu^2\, D_F(p) &= \delta^4(x' - x) \\ D_F(p) &= \frac{-1}{p^2 + i\epsilon}\left(g_{\mu\nu} - (1-\eta)\frac{p_\mu p^\nu}{p^2}\right)\end{aligned}$$

- **Electron/Positron** (Dirac fermionic spinor wave equation)

$$\begin{aligned}(\gamma_\mu \partial_\mu - m)\, S_F(p) &= \delta^4(x' - x) \\ S_F(p) &= \frac{1}{\gamma_\mu p_\mu - m + i\epsilon}\end{aligned}$$



Summary

- Introduced quantum engineering **design rules**
- **Unified formalism** based on QPI propagator
- Extended QPI to **entangled biphoton loops**
- Demonstrated the power of our quantum design rules by the resolving **complementarity paradox**
- Proposed lens **bifurcation** of coherent light
- Showed **Klein-Gordon** photon applicable in many quantum imaging and communication applications
- Future **CAD tools** to include full photonic QED effects