# Dirac in Australia

## Contents


1 Dirac Goes Bush 2

2 Who Took the Photos? 3

3 Up a Gum Tree 4

4 Dinner with Dirac 4

5 Dirac's Lectures 5

6 Dirac Monopoles 6

7 Dirac Number 6

8 Sommerfeld-Dirac Paradox 6

References 8


Paul Dirac was one of the early quantum mechanics who, among his many significant contributions, predicted the existence of antimatter in 1928, as a consequence of finding the correct relativistically invariant wave-equation for the electron. He also formulated the original many-body description of how light interacts with matter, viz., quantum electrodynamics.

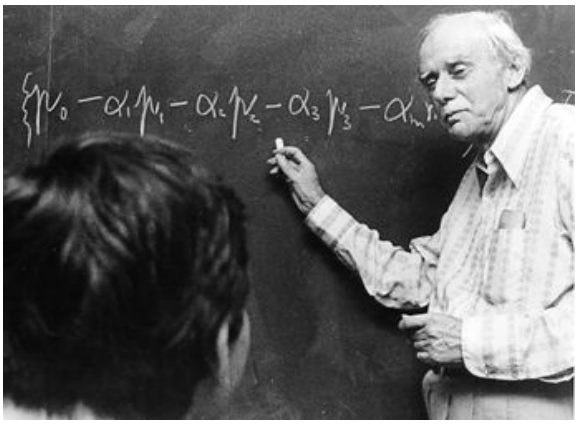


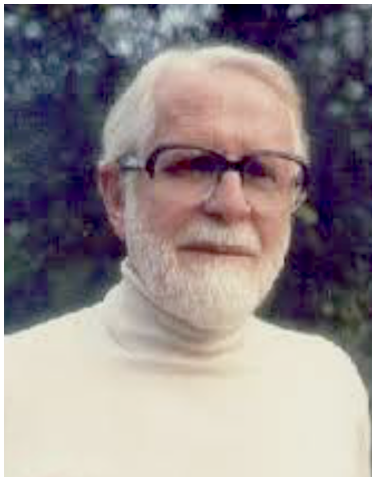
Figure 1: The man and his equation.

Although Dirac has not received the same degree of public awareness as say, Einstein or Feynman, biographical essays do exist [1] and several complete biographies have become available more recently [1, 2, 3, 4]. My review of *The Strangest Man* [3] was published by the New York Journal of Books in 2010.

# 1 Dirac Goes Bush

Dirac visited Australia in 1975. I don't recall why and he didn't come to Melbourne. My Master's thesis adviser, Prof. Christie Eliezer, was one of Dirac's few research students from 1941 to 1944 [4] so, he and I flew to Adelaide to see Dirac while he was visiting the Physics Department at the University of Adelaide under the auspices of Angas Hurst [5] and Bert Green [6]. See Figure 2.

(a) Prof. Angas Hurst.



(b) Prof. Bert Green).

Figure 2: Dirac's hosts at the University of Adelaide.

On the weekend, we went for a hike (at Dirac's request) and a BBQ lunch in the Adelaide Hills (Fig. 3). It was rather cold, so people had their jackets and coats on. Only Cambridge types would be caught dead wearing a suit and tie in the Australian bush.

(a) Dirac (left) and Eliezer (center).

(b) Here I am with Eliezer (2nd and 3rd from right).

Figure 3: Walkabout and BBQ in the bush.

I should also add that I was struck by how little was actually said between Dirac and Eliezer (see Figures 4). Although some words were no doubt exchanged, there was never any real conversation. Given Eliezer's very

affable nature and the various stories he had told me about working for Dirac, this looked really peculiar to me. In fact, a casual observer seeing this could only conclude that these two had never met before, let alone have a personal history together. But I was about to find out firsthand what was behind all that (see Section 4).

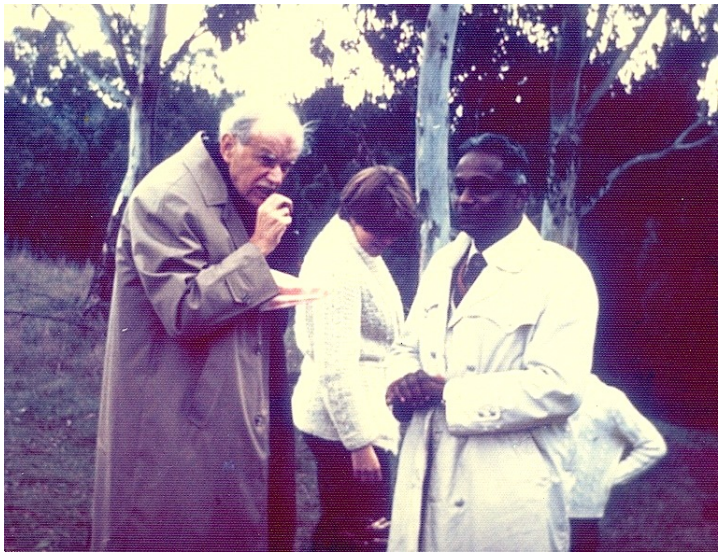


Figure 4: Dirac chewing on another tough problem.

## 2 Who Took the Photos?

Originally, I thought I had taken the photograph in Figure 3a, thus making me the "genius" behind the camera who forever lost the opportunity to say "This is me with Dirac!"

**Update of Feb 2010:** I now recall that, as a poor student, I didn't even own a camera at that time. Therefore, someone else must have taken that photo (along with Figs. 3b and 4). In addition, I now remember that I was with a group of people immediately in front of Dirac and, based on the angle, it looks like the photographer was in our group. The good news is, I'm not quite as dumb as I had originally thought.

**Update of Mar 2011:** Apparently, the unidentified person in Figure 3 is Prof. John Carver, who was then deputy chair of the Dept. of Physics at Adelaide University [7].

**Update of Aug 2012:** I finally discovered a photo of me at the same event. See Figure 3b. This clearly establishes that it was not me taking the photographs, but that I was present. Circumstances didn't put me in any shots with Dirac (or, at least, not any that I ended up possessing) but, by inference, being in a shot with Eliezer (and that inimitable white raincoat) does make the connection.

An online review of a new biography of Christie Eliezer [4] contains some remarkable photographs of both Eliezer and Dirac, as well as other interesting historical documents.

# 3 Up a Gum Tree

Dirac supposedly had a thing about trees. I had read somewhere that someone went to visit Dirac at his home in Cambridge and his wife (Wigner's sister) answered the door. When the visitor enquired if Dirac was at home, she said that he was home, but he was up a tree.

During our walk (see Figure 3a), I had some confirmation of this story. I was walking in front of Dirac with some other people when I suddenly heard something like an animal scurrying off to one side of the track. I assumed it was a rabbit or possibly a kangaroo. I turned around and discovered, to my great astonishment, that it was Dirac. He had taken off like a rocket up the slope (in his Sunday suite and shoes, mind you) toward a gum tree. Apparently, he was fascinated by a particular tree and wanted to get a closer look at it. I was very impressed at how he virtually sprinted up the hill, given that he was in his mid seventies!

Here is another explanation of Dirac's penchant for trees:

> *English physicist and Cambridge University professor Paul Dirac was an avid mountain climber and occasionally ascended such well-known peaks as Mount Elbruz in the Caucasus. In preparation for such excursions, Dirac would often climb trees in the hills just outside Cambridge - wearing the same black suit in which he was invariably seen around the university campus.*

# 4 Dinner with Dirac

A dinner party was thrown for Dirac at the home of Prof. Hurst. It was buffet style. While Dirac's wife (Figure 5) was holding court in the dining room (some of us thought she was quite the Brunhilde), Dirac snuck off into an adjacent, darkened room to eat his dinner. Since I was the only person who seemed to notice his stealthy departure, I soon decided to follow him and sat down beside him. I already knew from Eliezer, and reading other anecdotes, that Dirac did not do small talk. But I was now about to get schooled in the deeper significance of those warnings.

Figure 5: Margit and Paul Dirac.

To get around any silent treatment, I chose to avoid any small talk and cut to the chase by asking Dirac about black holes and singularities [8], I believe. No answer. After what seemed like minutes, still no answer. All of a sudden, it became impossible to know whether he hadn't heard the question, didn't have an answer (and was possibly embarrassed) or just didn't want to answer. It turned out to be none of these.

Small talk is a social lubricant, so it's extremely distressing not to get any response at all. You really don't know what is going on, because the expected natural flow has suddenly come to a screeching halt. Deciding what to do next is like trying to loosen a rusted nut—everything feels totally jammed up. And keep in mind, this is not just some old codger. He's one of the most eminent physicists of the century so, you also have that straightjacket to deal with. It's like falling off a bottomless cliff: you wish it would come to an end just to get rid of all the unbearable feelings. So, you don't know whether to keep trying harder or simply give up. Physicists don't usually give up easily.

While all this was swirling around in my head, I heard a small voice start up unexpectedly, after what seemed like eons and was quite possibly a minute or two. I eventually realized a series of very considered statements was now pouring forth. I suddenly realized it was Dirac responding to my question. I vaguely remember he seemed to be objecting to the notion of a black hole singularity (see item 2 below). By that time, however, I was so traumatized that I couldn't remember my own question, and I wasn't about to start the whole nerve-racking process over again. Sometimes, it's better to know when to give up.

More recently, I've come to learn about two additional things that may have been working against me:

1. Apparently, Dirac had digestive problems all his life [3] and it wasn't remedied until 1980. Trying to strike up a conversation while he was eating was possibly catalytic in bringing about the gut-wrenching verbal interaction.

2. It seems that Dirac was not too impressed with Stephen Hawking and originally voted against him as his replacement for the Lucasian chair [3]. If that's correct, my question about black holes may have been less than a stellar choice.

Perhaps I shouldn't feel too badly about the truncated dinner conversation. Dirac was Richard Feyman's hero. When they finally met at a conference, Feynman was keen to get Dirac's response to his path integral formulation of quantum mechanics, which Feynman had developed based on Dirac's quantum Action concept. Dirac's only response was, "That's interesting." and walked off [2].

# 5 Dirac's Lectures

Dirac gave three or four lectures while he was in Adelaide; all of which were quite boring (to me at the time) because he talked about things he had done forty or fifty years earlier without providing any personal insight into how he developed the ideas.

**Update of July 2013:** Apparently, the YouTube videos [9] of Dirac lecturing were recorded in 1975 while he was at the University of Canterbury in Christchurch, New Zealand. That's two additional places (UNSW and Canterbury) I didn't know he visited on this same trip.

What I find most striking about these lectures now is that nary a scratch is made on a blackboard; although he does make some hand gestures. The only things that are written on the blackboard are all symbols, just like his monograph on quantum mechanics [10]. Yet, it's clear from his notion of regarding the vacuum as an infinite sea of electrons, or the time it takes light to cross a proton [11], and other visualizations, that Dirac must have had a vivid style of thinking that was not too dissimilar to that of Einstein. Dirac was trained as an engineer, after all. I'm therefore left to conclude that these visualizations were private (as an aid to himself). They were used to organize his ideas but never materialized as physical illustrations in their own right. Like Gauss, it seems Dirac preferred to cover his tracks that would otherwise have shown how he reached his conclusions.

In the second video, on Quantum Electrodynamics, Dirac points out that the cutoff, imposed to avoid the well-known infinities that arise in certain integrals, is on the order of "1000 million volts" (or 1 GeV, as we would say today). This is clear evidence that he was not just a mathematician, but a real physicist (originally trained as an engineer), even though he goes on to discuss what constitutes "sensible mathematics" when it comes to QED renormalization.

# 6 Dirac Monopoles

Much more exciting than the lectures on ancient quantum mechanics came the news that Blas Cabrera, at Stanford, was claiming to have measured a Dirac magnetic monopole. If such a thing really existed, it would account for the constant magnitude of the electric charge. I remember Dirac was skeptical saying that magnetic monopoles should be lying around all over the place, if they existed. After phoning Luis Alvarez, the eminent experimental physicist at UC Berkeley, Dirac reported he didn't believe Cabrera's results. He was right, but a hint of irony remained: the inventor of the magnetic monopole didn't believe in his own invention.

Nonetheless, physics is an experimental science so, the search for magnetic monopoles has continued [12] but that may be about to come to an end [13].

# 7 Dirac Number

Prof. Eliezer (my M.Sc. adviser) was one of Dirac's very few research students; having worked under him in the early 1940s [4]. Since Eliezer worked with Dirac, he gets a Dirac number = 1 (if there was one) — by analogy with the Erdős number in mathematics — and I can claim, somewhat whimsically, Dirac number 2, since I worked with someone who worked with Dirac.

The reason Dirac had so few students was, in part, due to his taciturn nature. Actually, *taciturn* doesn't come close to capturing the effect it has on you. As I already described in Section 4), you really had to experience it directly to feel its almost overwhelming emotional impact.

There is a famous story about Leopold Infeld who dreamt of working with Dirac. When he finally got to meet him as part of an interview for a research position, he came away in tears because nothing was said and Infeld knew immediately he could never cope with that total lack of personal interaction.

# 8 Sommerfeld-Dirac Paradox

Eliezer was fond of reminiscing about his time with Dirac. Of the many stories he told me, the one that intrigued me the most, I always thought of as the *Eliezer Paradox*, although I later found out it was more widely known under other names like, the *Sommerfeld-Dirac Paradox*, Sommerfeld Puzzle, etc.

The story goes that Sommerfeld derived the eigenvalues corresponding to the fine structure of the hydrogen spectrum by incorporating relativistic corrections into the original Bohr planetary model of the atom. A beautiful account of this semi-classical approach can be found in Max Born's book [14], which was de rigueur reading and published right on the cusp of the arrival of the new *matrix mechanics* version of quantum theory [6]. Sommerfeld's work was published about a decade **before** Dirac derived his famous equation for the relativistic electron (see Figure 1) based on incorporating Pauli's concept of fermionic spin. The paradox is that the Sommerfeld solution contains no notion of electron spin. On the other hand, if you try to take

spin out of the Dirac equation, it all falls apart. So, how can these two seemingly disparate approaches be consistent with one another?

> *How wonderful that we are met with a paradox. Now we have some hope of making progress.*
> —Niels Bohr

Eliezer would have told me this story in particular, because he and I both knew that trying to resolve paradoxes of this type can often lead to quite deep results in physics (even decades later). However, although I was intrigued by this story, I never pursued it. If you're wondering why Dirac never pursued it either (or, certainly never published anything on it), he probably left it to others in the same way that he left Christie Eliezer to find the so-called *runaway solutions* to Dirac's classical extended-electron model [4, See essay by A. Pais].

Moreover, regarding Eliezer and me, I knew that neither of us was sufficiently up to speed on quantum field theory (I certainly wasn't at that particular time) so, I just assumed the Eliezer Paradox had most likely already been resolved in the context of more modern physics theories. I was wrong. To my amazement in recent years, I became aware that the Sommerfeld-Dirac Paradox is still being debated. Here is a brief selection of comments and claimed resolutions.

In 1942, Peter Bergmann (one of Einstein's assistants) [15] explained the paradox as being due to the cancelatiion of two errors.

> *"it so happens that the two errors—the neglect of the wave function and the neglect of spin—cancel each other in the case of the hydrogen atom"*

In 1968, Heisenberg [16] called the coincidence a miracle and continued:

> *"It would be intriguing to explore whether this is about a miracle or it is the group-theoretical approach which leads to this formula."*

In 1983, Biedenharn [17] claims

> *"a rotating frame makes it consistent"*

where the frame of reference rotation rate is synchronous with the precession rate.

In 1995, Weinberg [18] declared it to be:

> *"Absolutely chance coincidence."*

Even as recently as 2004, we find this doozy [19]:

> *"In the quasi-classical approximation, $L^2 = (l+1/2)^2\hbar^2$ is a correct result, so that if Sommerfeld 'did good science', then his formula would contain $l + 1/2$ instead of $n_\phi$."*

There are surely similar, more recent, claims ongoing. I stopped looking.

For my part, I had become less persuaded by either Eliezer's original (good) suggestion or Heisenberg's 1968 comment, that the paradox might be resolved by an analysis of dynamical symmetries [20]. For one thing, the SO(4) symmetry of the Pauli hydrogen atom [21] is broken by relativistic modification of the Coulomb potential, thus lifting the $(2l + 1)^{-2}$ spectral degeneracy. But that also leaves only the geometrical symmetry,

i.e, the usual angular momentum states of SO(3). It's not clear how that smaller symmetry can support a group-theoretic derivation of the more complicated formula for the relativistic energy eigenvalues.

All of the above notwithstanding, at long last—as a kind of postscript dedicated to Prof. Eliezer—my rather different resolution of the Sommerfeld-Dirac paradox [22] was finally presented at the American Physical Society global conference in 2025. (Never give up!)

# References


[1] Abdus Salam and Eugene Wigner (eds.), *Aspects of Quantum Theory*, Cambridge Uni. Press (1972)

[2] Helge Kragh, *Dirac: A Scientific Biography*, Cambridge Uni. Press (2005)

[3] Graham Farmelo, *The Strangest Man: The Hidden Life of Paul Dirac*, Basic Books (2009)

[4] Ranee Eliezer, *Conquering Scientist: A Biography of Emeritus Professor Christie J. Eliezer AM 1918-2001*. La Sha Prints Pty. Ltd, (2012)

[5] C.A. Hurst, "Perturbation expansions in quantum field theory," Reports on Mathematical Physics, **57**, 1, 121–129, (2006)

[6] H. S. Green, *Matrix Mechanics*, Nooedhoff (1965)

[7] Peter Lyster, *private communication*, March 5 (2011)

[8] R. Penrose, "Gravitational Collapse and Space-Time Singularities," Phys. Rev. Lett. 14, 57, Jan 18, (1965). S. Hawking, *Properties of Expanding Universes*, Ph.D. thesis, U. Cambridge, (1966)

[9] Videos of Dirac's lectures have now been posted on YouTube c.2013

1. Dirac Lecture on Quantum Mechanics.
2. Dirac Lecture on Quantum Electrodynamics.
3. Dirac Lecture on Magnetic Monopoles.
4. Dirac Lecture on his Large Numbers Hypothesis.

[10] P. A. M. Dirac, *Principles of Quantum Mechanics*, Oxford University Press (Orig. 1930. Last revision 1967)

[11] Dirac's large numbers hypothesis: *The current age of the universe, divided by the time it takes light to cross the radius of a proton, is about* $10^{40}$*. This number is also approximately equal to the ratio of the strengths of the electromagnetic and gravitational forces. Dirac felt that the approximate equality of these two large numbers was too unlikely to be accidental and that some physical process must be at work to maintain the equality. Since the first number clearly changes in time (because the age of the universe is increasing), Dirac proposed that the "fundamental constants of nature" entering the second number should also change in time, to maintain the equality.* [Source. Scroll down to alphabetic entry: Dirac Large Number Hypothesis.]

[12] Ongoing search for magnetic monopoles: "Theoretical and Experimental Status of Magnetic Monopoles," http://arxiv.org/abs/hep-ex/0602040v1 (2006), "Search for magnetic monopoles in polar volcanic rocks," http://arxiv.org/abs/1301.6530 (2013)

[13] Physicists discover hidden aspects of electrodynamics (phys.org, April 11, 2017). The basic claim is

1. Maxwell's theory displays a symmetry called the EM duality.
2. However, while electric charges exist, magnetic charges have never been observed in nature.
3. If magnetic charges do not exist, the EM symmetry cannot exist.
4. Gravity plus quantum effects disrupt the EM duality. Hence, magnetic monopoles don't exist.

[14] M. Born, *Mechanics of the Atom*, Ungar, 1960 (Orig. German 1924. Orig. English 1927)

[15] P. G. Bergmann, *Introduction to the Theory of Relativity*, Dover 1976 (Orig. pub. 1942)

[16] W. Heisenberg, "Ausstrahlung von Sommerfelds Werk in der Gegenwart," Physikalische Blätter, 24 530-537 (1968)

[17] L.C. Biedenharn, "The 'Sommerfeld Puzzle' Revisited and Resolved," Foundations of Physics, vol. 13, no. 1 (1983)

[18] S. Weinberg, *The Quantum Theory of Fields*, Vol. 1: Foundations, Cambridge Univ. Press (1995)

[19] Ya. Granovskii, "Sommerfeld formula and Dirac's theory," Uspekhi Fiz. Nauk, 47(5) 523-524 (2004)

[20] NJG, "Unitary declension of dynamical symmetries for the time-dependent harmonic oscillator," J. Math. Phys. 20, 1539 (1979)

[21] W. Pauli, "On The Hydrogen Spectrum From the Standpoint of The New Quantum Mechanics," Z. Physik 36, 336, (1926)

[22] NJG, "The Sommerfeld-Dirac Paradox Reexamined," APS Global Summit, Anaheim, California (2025)